

B.Sc. Semester - 3 (CBCS) Examination
Nov/Dec. -2021 [NEW COURSE]
Mathematics-M301(Core)

Time: 2:30 Hours

Marks: 70

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q-1

(A) ATTEMPT ANY TWO:

10

(1) Define: Vector Space.

Prove that $-v$ is unique $\forall v \in V$ where V is a vector space.

(2) Define: Subspace of a vector space.

If W_1 & W_2 be any subspaces of a vector space V , then prove that $W_1 \cap W_2$ is also a subspace of V . Will $W_1 \cup W_2$ be a subspace always? Justify your answer.

(3) Define: Linearly Independent Set & Linearly Dependent Set

Prove that a subset of a linearly independent set is also linearly independent.

(B) ATTEMPT ANY ONE :

04

(1) Show that the set $V = \{(x, y) \in \mathbb{R}^2 / y > 0\}$ is a vector space over $F = \mathbb{R}$ under the vector addition and scalar multiplication defined as follows.

$$(a, b) + (c, d) = (a + c, bd), \forall (a, b), (c, d) \in V$$

$$\text{and } \alpha(a, b) = (\alpha a, b^\alpha), \forall (a, b) \in V, \forall \alpha \in \mathbb{R}.$$

(2) Check linear dependence of the following sets:

$$(A) \{e^{2x}, e^{4x}, e^{6x}\}$$

$$(B) \{\sin(-x), \cos(x), \sin\left(x + \frac{\pi}{3}\right)\}$$

Q-2

(A) ATTEMPT ANY TWO:

10

(1) State and prove the extension theorem.

(2) Let V be a finite dimensional vector space .Then, prove that for every subspace W_1 of V , there exists another subspace W_2 of V such that $V = W_1 \oplus W_2$.(3) If W_1 & W_2 be any two subspaces of a finite dimensional vector space V , then prove that

$$\dim(W_1 + W_2) = \dim(W_1) + \dim(W_2) - \dim(W_1 \cap W_2).$$

(B) ATTEMPT ANY ONE :

04

(1) Show that the set $\{1 - x, 1 + x, 1 - x^2\}$ is a basis of $P_2(\mathbb{R})$. Also find the coordinates of the vector $1 + 2x + 3x^2$ w.r.t. this basis.(2) Let $W_1 = \{(a, b, c, d) / a + c - d = 0\}$ & $W_2 = \{(a, b, c, d) / a + 2b = 0\}$ are subspaces of the vector space $\mathbb{R}^4$.Find $\dim(W_1)$, $\dim(W_2)$ & $\dim(W_1 \cap W_2)$. Also, deduce that $W_1 + W_2 = \mathbb{R}^4$.

Q-3

(A) ATTEMPT ANY TWO:

10

(1) Define: Gradient

If ϕ and ψ are differentiable scalar functions, then prove the followings:

$$(i) \nabla(\phi\psi) = \phi\nabla(\psi) + \psi\nabla(\phi) \quad (ii) \nabla\left(\frac{\phi}{\psi}\right) = \frac{\psi\nabla(\phi) - \phi\nabla(\psi)}{\psi^2}$$

(2) Define: (i) Divergence (ii) Curl

If $\bar{f}$ be a function whose second order derivative exists, then show that $\text{div}(\text{Curl}(\bar{f})) = 0$.

(3) Define: Laplacian Operator

Prove that, $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$ where $r = |\bar{r}|, \bar{r} = (x, y, z)$.

(B) ATTEMPT ANY ONE:

04

(1) If $(x, y, z) = x^2y + xy^2 + z^2$, then find $\nabla\phi$ and find unit normal at $(1, 2, 3)$.

(2) Show that the function $H = e^{-\lambda x}(c_1 \sin(\lambda y) + c_2 \cos(\lambda y))$ satisfies the Laplace equation where λ, c_1, c_2 are constants.

Q-4

(A) ATTEMPT ANY TWO:

10

(1) Define: Double Integration

Explain evaluation of double integrals and also explain change of order of integration.

(2) Define: Triple Integration

Explain change of Cartesian coordinates into cylindrical and spherical coordinates in triple integration.

(3) Evaluate: $\iiint (a^2b^2c^2 - b^2c^2x^2 - c^2a^2y^2 - a^2b^2z^2) dx dy dz$ over R, where R is ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

(B) ATTEMPT ANY ONE:

04

(1) Evaluate: $\iint xy dy dx$ over R,

Where, R is the third quadrant of the circle $x^2 + y^2 = 9$.

(2) Evaluate: $\int_0^a \int_0^b \int_0^c (x + y + z) dx dy dz$.

Q-5

(A) ATTEMPT ANY TWO:

10

(1) State and prove the Green's theorem.

(2) State Gauss divergence theorem and verify the theorem for $F = (4x, -2y^2, z^2)$ where R is $x^2 + y^2 = 4, z = 0$ & $z = 3$.

(3) Prove that $\beta(p, q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$, where $p > 0$ & $q > 0$.

(B) ATTEMPT ANY ONE:

04

(1) Verify the Stoke's theorem for $F = (2x - y, -yz^2, -y^2z)$ & S is the upper half of the unit sphere.

(2) Find the value of $\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)$.
